



# Bayes for Flexibility in Urgent Times

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# Big Picture

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Background

Freq&Bayes

Needed  
Probabilities

Fully  
Sequential  
Trials

Summary

- Efficacy is not a hypothesis; it is a matter of degree
- Hypothesis testing and associated thresholds have hurt science
- Would you rather know the chance of making an assertion of efficacy when the treatment has no effect, or the chance the treatment is effective?
- Probabilities conditioning backwards in time/information flow are not directly actionable



# Problems We Face in COVID-19

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Summary

- Relatively little known about the virus and its treatment
- Rapid decision making required
- Studies must be launched and concluded quickly
- DSMBs must act on information from other studies
- DSMB meetings cannot be scheduled far in advance
- A data look may be required at any time
- Best to just allow for infinitely many data looks
- See [hbiostat.org/proj/covid19](https://hbiostat.org/proj/covid19)



# High Level View of Statistical Approaches

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- Frequentist: probability of data given an assertion is true
- Bayesian: probability assertion is true given the data
- Frequentist type I error: probability of making an assertion of efficacy over the long run of replicate studies like yours **except** that the treatment has zero effect and does no harm
- Bayesian posterior probability of efficacy: probability of true efficacy underlying the process generating **our** data (probability that an assertion of efficacy is true)
- One minus posterior probability of efficacy: probability of no effect or harm (*regulator's regret*)



# What is Actionable?

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## What is Not Actionable

After a patient has a diagnostic test, the sensitivity and specificity of the test

## What is Actionable

The probability the patient has the disease



# Advantages of Bayes, *continued*

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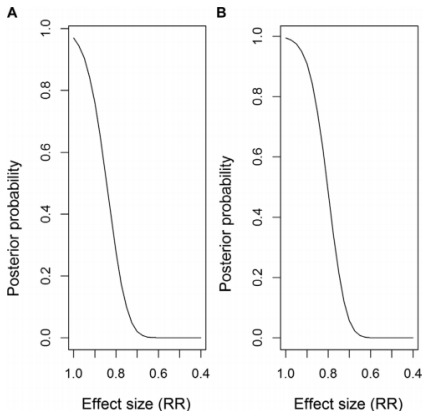
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**Figure 3** Posterior probability of specified effect sizes, using (A) flat prior, (B) evidence-based priors. The line shows the probability of the relative risk (RR) being lower than the values on the x-axis (ie, a bigger treatment effect). A RR <1 indicates that the primary outcome rate is smaller in the intervention arm compared with the control arm.



# Examples of Posterior Probabilities

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- Prob(any assertion or combination of assertions)
- Prob(efficacy  $> 0$ )
- Prob(efficacy  $> \text{MCID}$ )
- Prob(non-inferiority)
- Prob(efficacy  $> 0$ ) on Nov. 2: interpretation completely unaffected by:
  - Prob(efficacy  $> 0$ ) on Nov. 1
  - Flexibility:  
Prob(hit any 2 of 4 migraine headache endpoints)



# Besides The Interpretation Does It Matter That $p$ -values are Backward Probabilities?

- Can't inject appropriate skepticism into the calculation
- Can't inject prior relevant information (skeptical or positive) into the calculation
- Being backwards is the **cause** of multiplicity problems: Multiplicity is caused by the chances you give data to be extreme, **not** from the chances you give assertions to be true
- Being backwards means you have to take into account how the data arose instead of just interpret the data at hand
  - Frequentist approach is cumbersome for flexible sequential designs or adaptive designs





# Fully Sequential Trials: Continuous Learning with Unlimited Looks

(In a Bayesian analysis) It is entirely appropriate to collect data until a point has been proven or disproven, or until the data collector runs out of time, money, or patience. - Edwards, Lindman, Savage (1963)

- Run 50,000 **different** clinical trials  
(differ on amount of efficacy)
- For each, sample one  $\mu$  (true efficacy) from the prior
- Generate data ( $n = 500$ ) under this truth
- Do analysis after 1, 2,  $\dots$ , 500 subjects studied ( $\leq 500$  looks)
- Stop the instant  $\text{Prob}(\mu > 0) \geq 0.95$  (efficacy) or  $\text{Prob}(\mu < 0.05) \geq 0.90$  (futility)
- See [fharrell.com](http://fharrell.com) for details and code
- See [hbiostat.org/proj/covid19](http://hbiostat.org/proj/covid19) for a detailed sequential COVID-19 clinical trial plan



# Sequential Testing Simulation, *continued*

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- 20393 trials stopped early for efficacy
- 28438 trials stopped early for futility
- 1169 trials went to completion ( $n = 500$ )
- Average post. prob. of efficacy at stopping for efficacy: 0.961
- Of trials stopped early for efficacy, proportion with  $\mu > 0$ : 0.960
- Average post. prob. of futility at stopping for futility: 0.920
- Of trials stopped early for futility, proportion with  $\mu < 0.05$ : 0.923



# Calibration of Posterior Mean at Stopping for Efficacy

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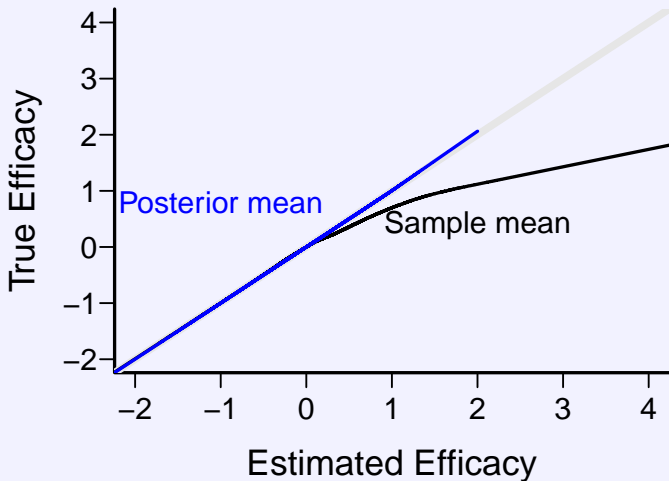
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# Advantages of Bayesian Approach

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- Computes probabilities on the actionable scale
- Is based solely on basic laws of probability; no special recipes needed
- Is flexible without hurting the science
- Allows experimentation until sufficient evidence
- Use in complex adaptive designs no more complex than use in simple static design
- Can provide simultaneous totality of evidence
- Non-inferiority involves just another posterior probability
- Evidence for non-trivial effects  $P(E > \epsilon)$
- $P(\text{hitting complex efficacy targets})$
- Math for incorporating external information



# What is the Greatest Hesitance to Adopting Bayes?

- Fear of not preserving type I assertion probabilities (there is no type I “error”)
- Type I assertion probability = long-run  $P(\text{assertion of efficacy})$  **if efficacy = 0**
- This probability is **independent of the data**
- Contrast with  $P(\text{mistake} \mid \text{data}) = 1 - \text{posterior } P(\text{efficacy})$
- Type I error **is not regulator’s regret** (approving a drug that doesn’t work) but is an assertion probability **assuming the drug doesn’t work**
- Analogy:
  - Judging a politician by how often he talks vs.
  - Judging him by how often he tells the truth



# New Resource and Discussion Board

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- Introductory Bayesian design and analysis course:  
[hbiostat.org/doc/bayes/course.html](https://hbiostat.org/doc/bayes/course.html)
- Bayesian  $t$ -tests: [hbiostat.org/doc/bbr.pdf](https://hbiostat.org/doc/bbr.pdf) Chapter 5
- Discussion board for the presentation you are viewing:  
[bit.ly/datamethods-whybayes](https://bit.ly/datamethods-whybayes)
- [hbiostat.org/proj/covid19](https://hbiostat.org/proj/covid19)