A COMPARISON BETWEEN THE PROPORTIONAL ODDS AND CONTINUATION RATIO MODELS FOR ANALYZING ORDINAL OUTCOMES

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This talk will begin with an overview of the proportional odds (PO) and continuation ratio (CR) ordinal logistic models. Then it will outline how assumptions of these two models are checked. It will also compare the two models in the context of developing a clinical prediction model for diagnostic outcomes of bacterial infections in young infants enrolled in a World Health Organization/ARI Programme study in developing countries. Residual plots will be presented for assessing linearity and distributional assumptions. The ease with which the CR model can be relaxed to allow for X by Y interaction will be exploited to allow flexibility in the model. An example will be given where penalized maximum likelihood estimation is used to keep the more flexible model from overfitting. Sample S–PLUS code and a list of references for PO and CR models will be provided.

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Joint Statistical Meetings
Chicago IL

6 Aug 96

Outline

- Ordinal Response variable Y
- Proportional odds (PO) and continuation ratio (CR) models
- Score residuals for checking PO assumption
- Partial residuals for checking PO model
- Partial residuals for checking CR model
- Example: WHO/ARI Multicentre Young Infant Study

Ordinal Response Variable Y

- Only rank ordering matters (spacing irrelevant)
- Examples: severity of pain, quality of life, severity of clinical endpoint
- Y coded $0, 1, \ldots, k$

PO Model

- Special case: Wilcoxon test

$$\Pr[Y \geq j|X] = \frac{1}{1 + \exp[-(\alpha_j + X\beta)]},$$

$j = 1, 2, \ldots, k$.

- Fixed j: binary logistic model for $Y \geq j$

CR Model

- Categorical Cox model

$$\begin{aligned}
\Pr(Y = j | Y \geq j, X) &= \frac{1}{1 + \exp[-(\theta_j + X\gamma)]} \\
\text{logit}(Y = 0 | Y \geq 0, X) &= \text{logit}(Y = 0 | X) \\
&= \theta_0 + X\gamma \\
\text{logit}(Y = 1 | Y \geq 1, X) &= \theta_1 + X\gamma \\
&\ldots \\
\text{logit}(Y = k-1 | & \\
Y \geq k-1, X) &= \theta_{k-1} + X\gamma.
\end{aligned}$$

- Fixed j: binary logistic model for $Y = j | Y \geq j$

Score Residuals for Checking PO

- Compute first derivative of log–likelihood with respect to each parameter, compute mean stratifying on Y

- Analogous to Schoenfeld residual in Cox model

- Plots U–shaped if PO holds

- Instead use fitted PO model to predict series of events $Y \geq j, j = 1, 2, \ldots, k$

- Compute score residuals from binary logistic model

$$U_{im} = X_{im}([Y_i \geq j] - \hat{P}_{ij})$$

- Get mean and CL for each column of U

Partial Residuals for PO Model

- Again use residuals for binary model, for each cutoff j and the m'th predictor

$$r_{im} = \hat{\beta}_m X_{im} + \frac{[Y_i \geq j] - \hat{P}_{ij}}{\hat{P}_{ij}(1 - \hat{P}_{ij})},$$

- Smooth, e.g. `lowess`

- Check for parallelism across j

- Also check linearity in the logit

Partial Residuals for CR Model

- Again use binary logistic model partial residuals
- Separately fit sequence of binary models with applicable (increasingly small) subsets of subjects
- Parallelism = constant slope assumption OK

Example: WHO/ARI Multicentre Study of Clinical Signs and Etiological Agents of Pneumonia, Sepsis, and Meningitis in Young Infants

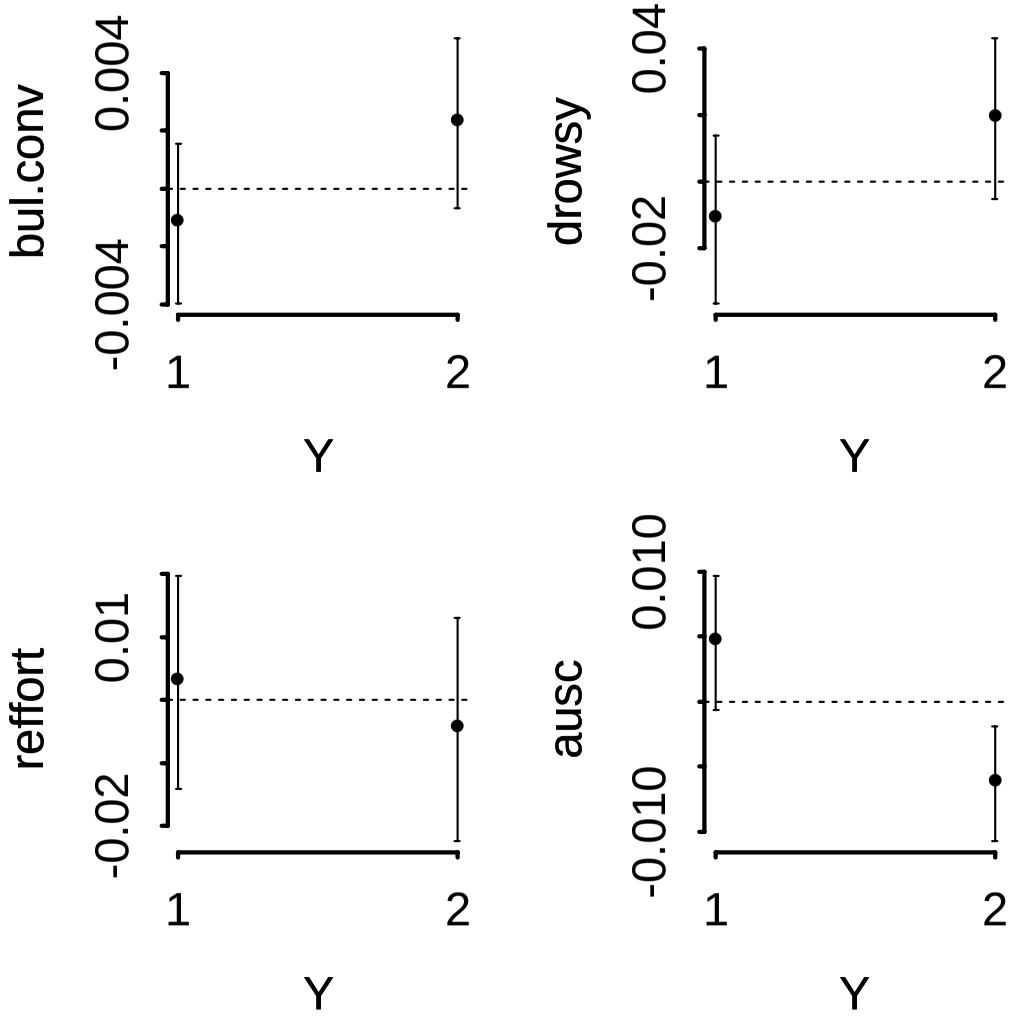
- Goal: Improve diagnosis/treatment of infants < 3 mos. old in developing countries
- Diseases of interest: Serious bacterial infection (sepsis, meningitis, pneumonia)
- $n = 4552$, 4 countries
- 14 symptom/sign summary scores, 4 vital signs

Response Variable

Y	Definition	Frequency
0	Neither	3551
1	Mild hypoxemia or Pneumonia by X–ray	490
2	Severe hypoxemia or + Bacterial blood culture or + Cerebrospinal culture	511

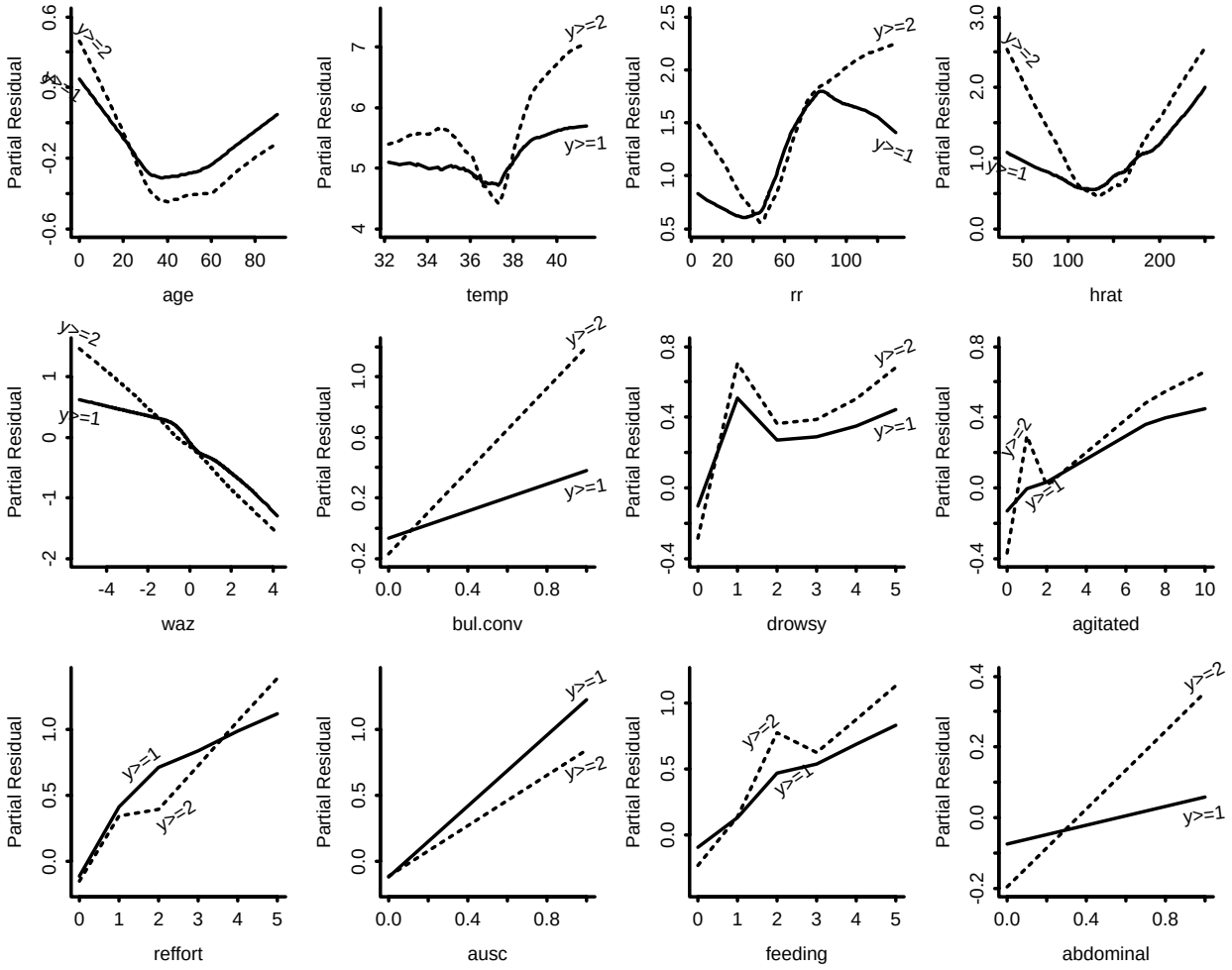
Checking PO Assumption (Assuming Linearity)

- Binary model score residuals



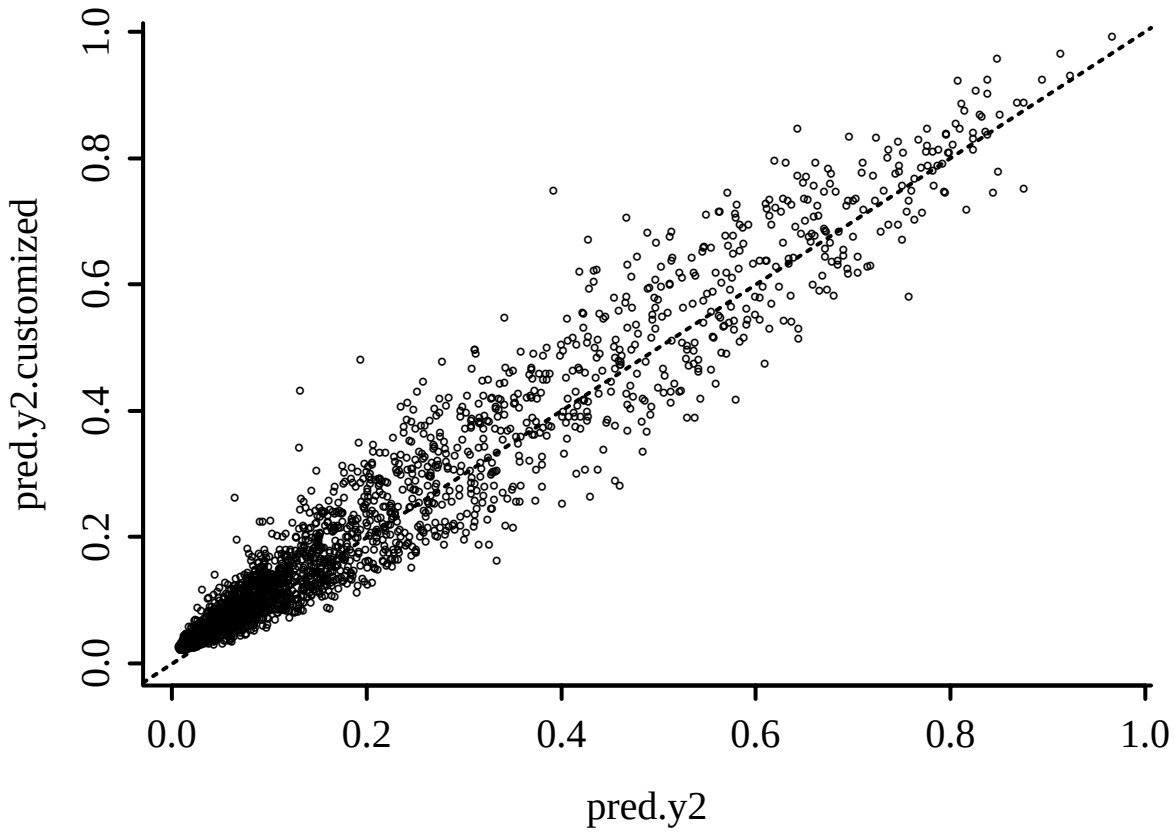
Checking Transformations and PO Assumption

- Series of binary model smoothed partial residual plots



What is Effect of Non-PO?

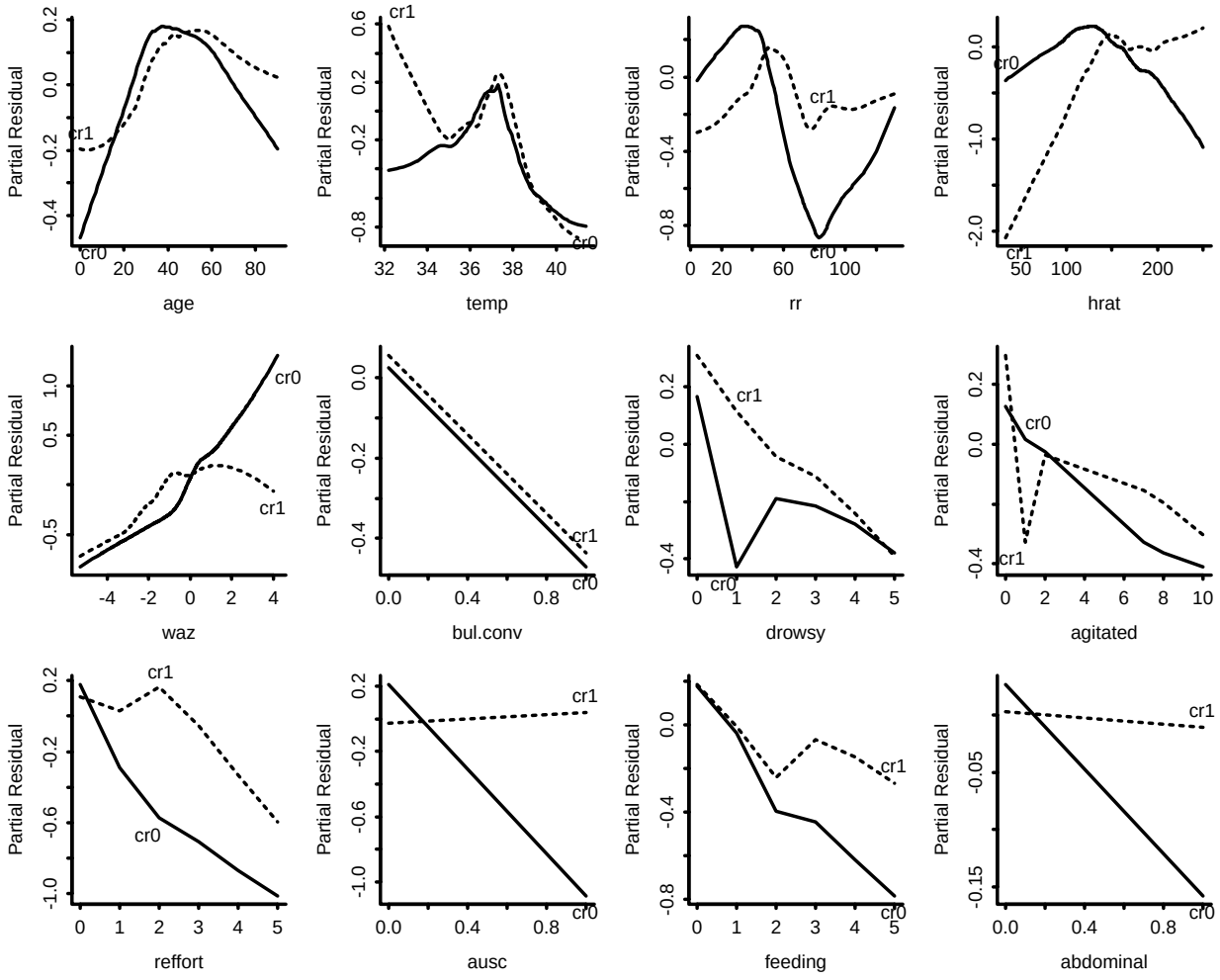
- Fit separate binary model for Prob($Y = 2$)
- Compare with predictions from PO model



Checking Transformations and CR Assumptions

- Series of binary model smoothed partial residual plots

- Each one for Prob($Y = j | Y \geq j$)


Partial Residual
age
temp
rr
hrat
waz
bul.conv
drowsy
agitated
reffort
ausc
feeding
abdominal
cr0
cr1

Extended CR Model

- Some or all of the X's can have different regression coefficients for some or all of the Y cutoffs

$$\Pr(Y = j | Y \geq j, X) = \frac{1}{1 + \exp[-(\theta_j + X\gamma_j)]}$$

- Flexibility to fit any ordinal or polytomous model

- Best to state model as having main effects plus terms interacting with Y (j)

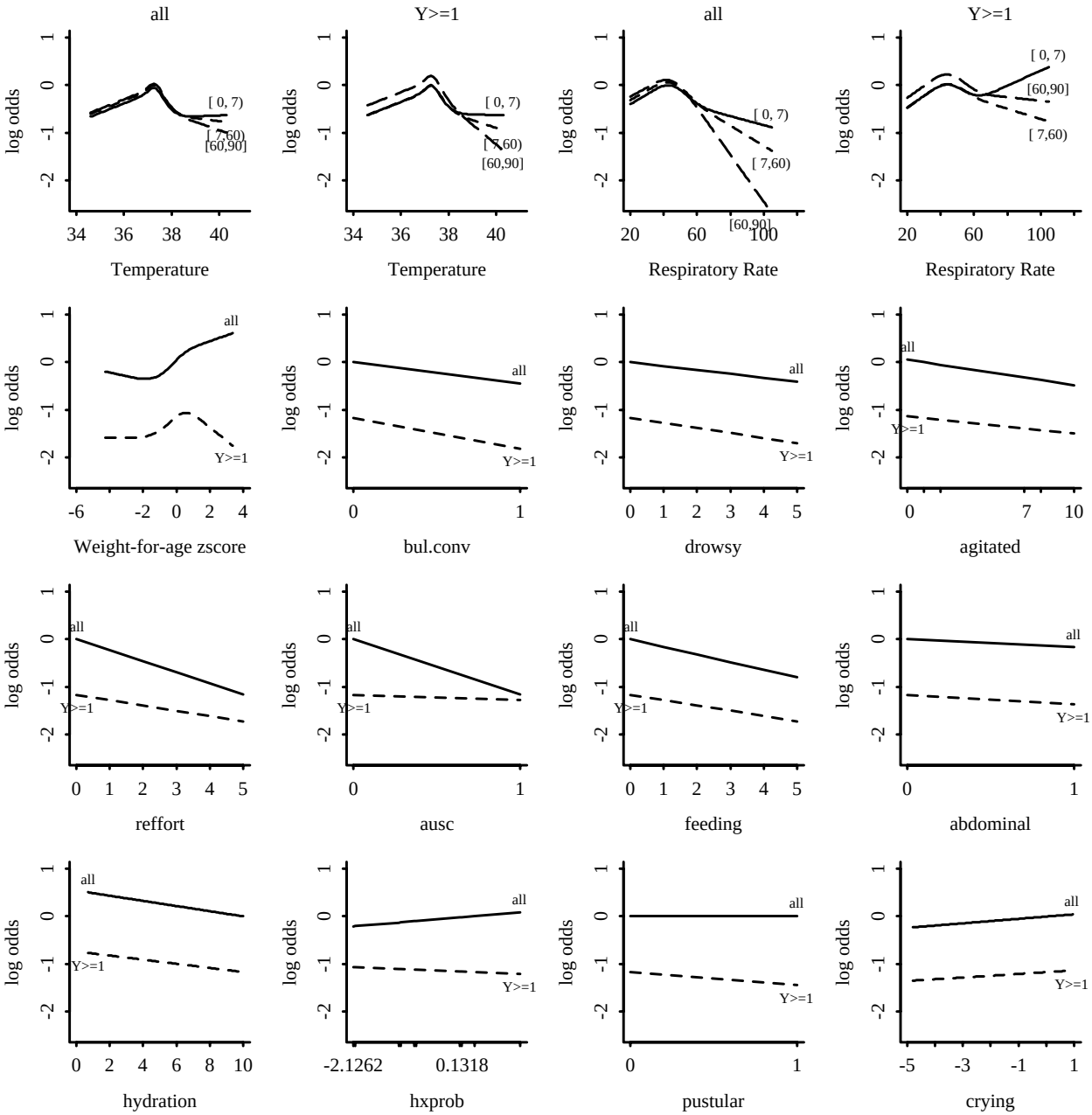
- Formal test for equal slopes assumption

- Can overfit, causes model's predictions to not be well calibrated

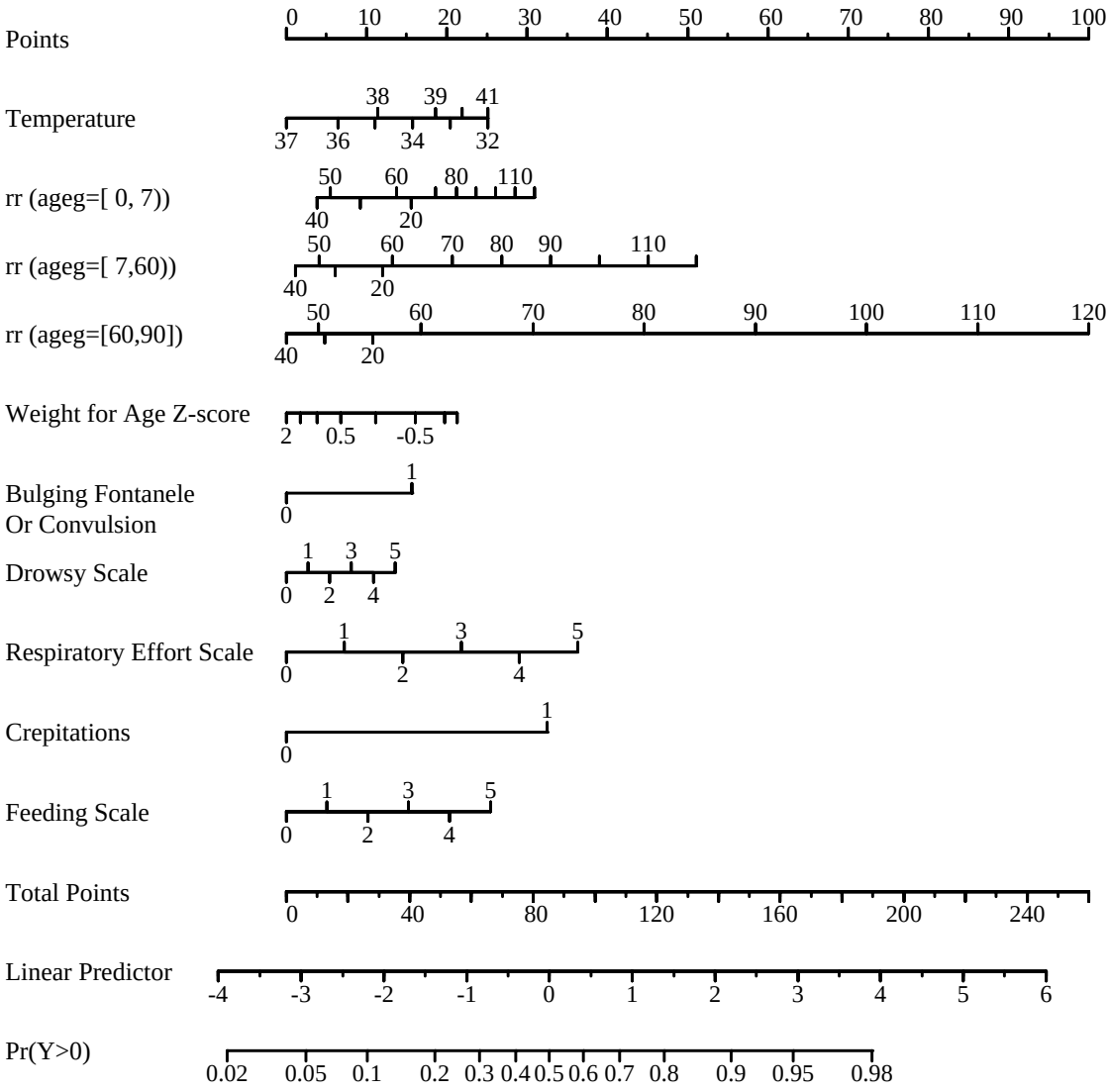
- Use penalized maximum likelihood estimation, penalizing primarily on the interaction terms (Gray, Verweij et al.)

- Allow for unequal slopes only as far as the information content in the data will support

Fits from Penalized Extended CR Model



Final Model: Prob($Y > 0$) Portion



Summary

- Smooth partial residual plots are useful for checking both linearity and constant slope assumptions

- Neither PO nor CR model fit the WHO/ARI dataset

- CR model easier to generalize

- Use differential penalization of $X \times Y$ interactions to achieve parsimony while fitting the data

"all" refers to prediction of $Y = 0$, and "$Y >= 1$" refers to the prediction of $Y = 1$ cond. on $Y \geq 1$.